

Advanced Field Balancing

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Advanced Field Balancing Techniques

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Introduction

Vibration in rotating machinery is commonly the result of mechanical faults including mass unbalance, coupling misalignment, loose components, and many other causes. Improving the levels of vibration should always include elimination of the source of vibration and not addressing the symptom by making balance corrections.

Mass unbalance will produce vibration due to the force generated by the eccentric weight. This force will be imposed at the running speed of the shaft, and depends on the amount of eccentric mass m , the eccentricity of the weight e_u , and the frequency of rotation ω . In more common terms the unbalance is defined by the eccentric weight, mounting radius, and shaft speed. The observed vibration signature will show elevated amplitudes at 1xRPM and no other significant frequencies when rotor unbalanced is the main fault.

$$F_{unbalance} = ms_u \omega^2 = \frac{\text{Weight, Oz}}{16 \frac{\text{Oz}}{\text{Lb}} \times 386 \frac{\text{in}}{\text{sec}^2}} \times \text{Radius, inches} \times \left(\frac{\text{RPM} \times 2\pi}{60 \frac{\text{sec}}{\text{min}}} \right)^2$$

Unfortunately, other common faults can also generate high levels of vibration at 1xRPM including coupling misalignment, looseness, rotor bows, and a variety of other sources. In some cases, these faults will produce other symptoms that can suggest corrections other than balancing should be done. Yet in many cases, balancing may be the chosen course of action for lowering vibration amplitudes even though it is not the source of vibration.

Once it is determined that balance corrections should be made, the balancing process includes measuring reference vibration, adding trial weights, observing the response due to trial weights, and using the response characteristics to determine the location of balance correction weights to reduce vibration to an acceptable level. Once a field balance has been completed on a machine (or similar machine) the response data from adding trial weights can be used to calculate future balance corrections using a one shot method (no trial weights required).

This paper provides a description of the process used for field balancing of a variety of rotors including single and multi-plane methods. Details include a theoretical description along with some practical suggestions, pitfalls, and examples for each of the balance methods.

Determining the Need for Field Balancing

Prior to attempting any balance corrections, a proper vibration analysis should be done to determine the likelihood that the machine is in fact out of balance. Making balance corrections to a machine with some other fault can in many cases reduce the vibration amplitudes. However, if balance corrections are made to a machine that is not out of balance to start with, the forces generated by the fault will still exist even though balance corrections may reduce the amplitude at some measurement points.

Unbalance will generate forces and corresponding vibration response at 1xRPM. For example, if the rotor speed is 3600 RPM, unbalance will produce vibration at 3600 CPM (60 Hz). The FFT plot in figure 1 shows an example of an unbalanced rotor, with vibration shown at the shaft speed and minimal vibration content elsewhere. Since this data was recorded from installed proximity probes and some minor imperfections exist in the shaft surface, there is a small 2xRPM component due to runout.

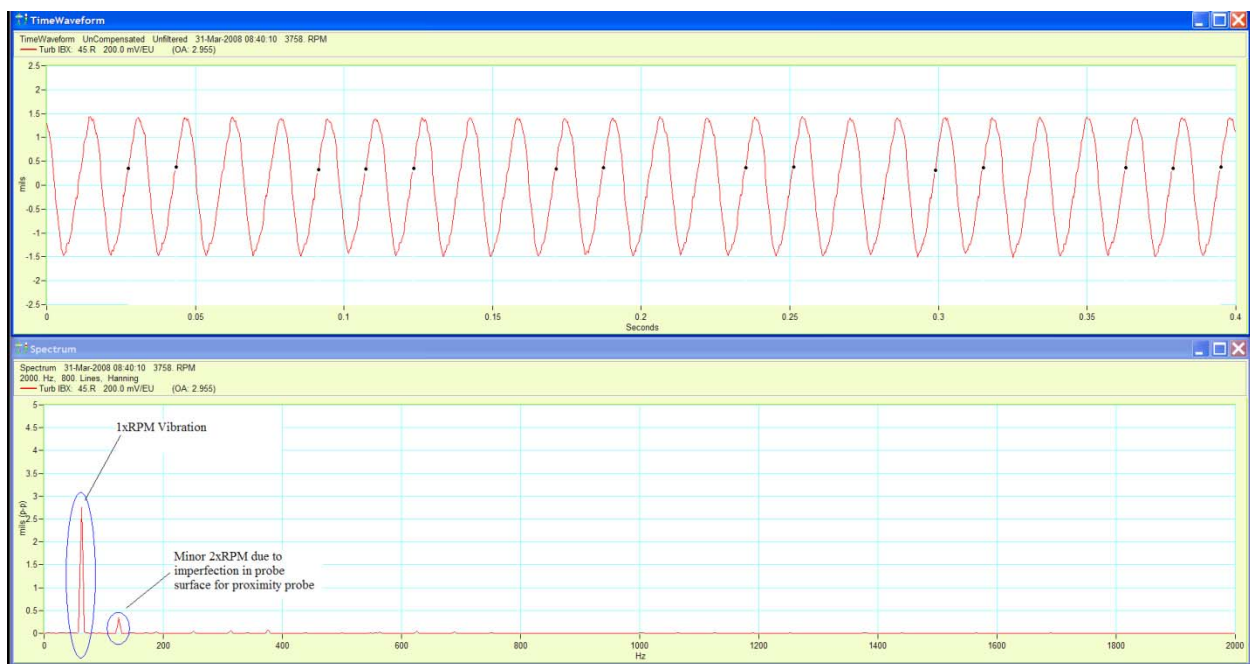


Figure 1 - Typical FFT from Unbalance

Another common characteristic of unbalance is the vibration phase difference that should be about 90° between vertical and horizontal locations, with the phase angles increasing with rotating as shown on the orbit plot in figure 2.

This display shows that the phase difference between two orthogonal probes (90° apart) is nearly 90°. The actual phase difference is 70° as shown in the notes on the plot which can occur due to a variety of reasons. Most common machinery with horizontal rotors will also show higher amplitude in the horizontal direction compared to vertical. Fans and motors will generally have a vertical/horizontal difference between 1:1 and 1:10 (horizontal possibly 10 times higher than vertical), which is usually dependant on the flexibility of the bearing housing and/or pedestals. This indicates that most equipment will be stiffer vertically than horizontal, which makes sense since vertical motion will be

resisted by tension/compression of the support where horizontal results in bending of the support. If an orbit plot is viewed using seismic probes (accelerometers) on a bearing housing, it is not unusual to see a large horizontal movement and little vertical movement as shown in figure 3.

The phase difference from one bearing to the other (inboard to outboard) can be used to assess if the unbalance is static or dynamic. In many cases static unbalance, indicated by the same phase angle in the same measurement direction at both bearings, can be corrected using a single plane balance method. When phase angles differ from one bearing to the other, multiple balance correction planes are often required.

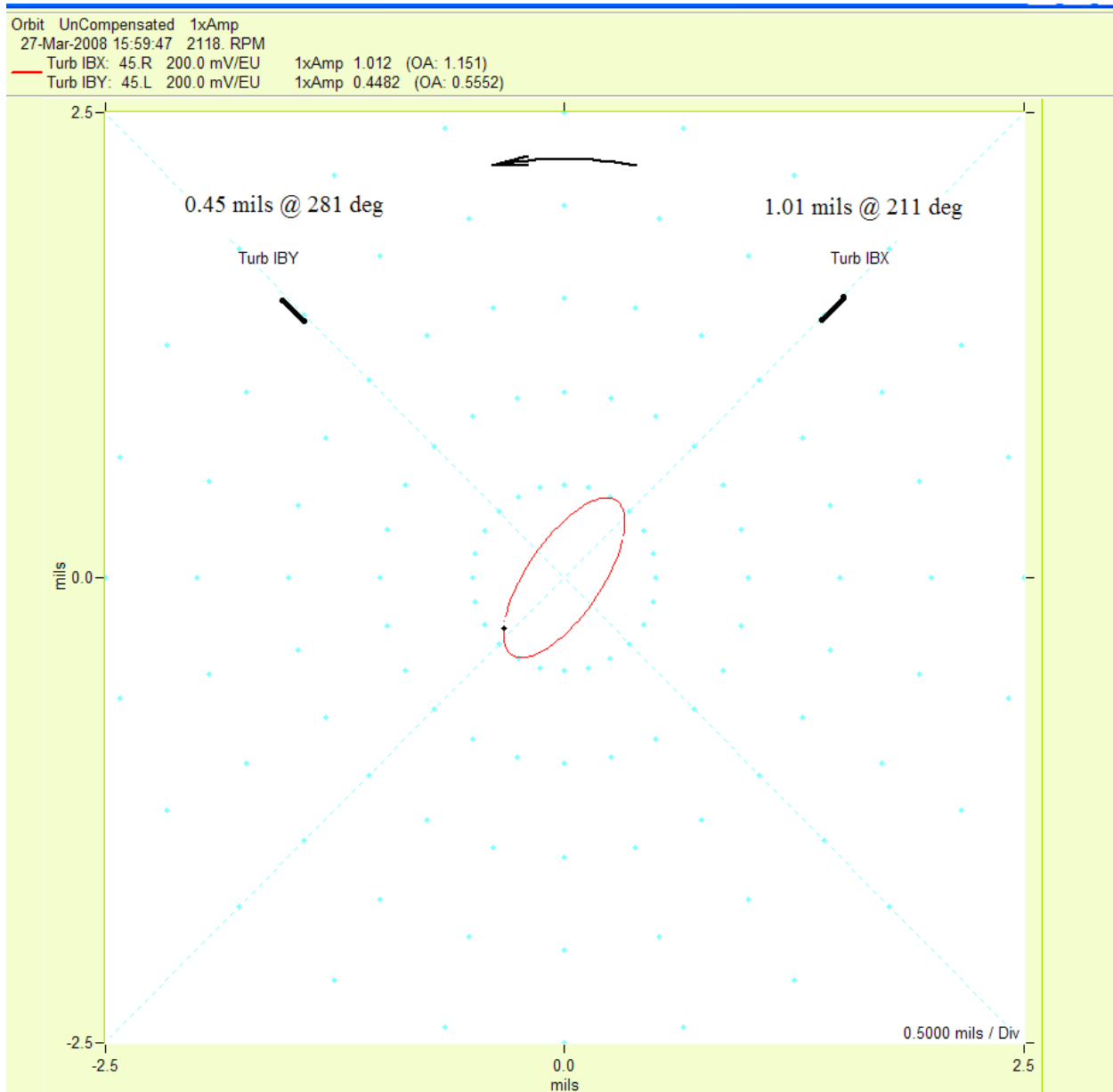
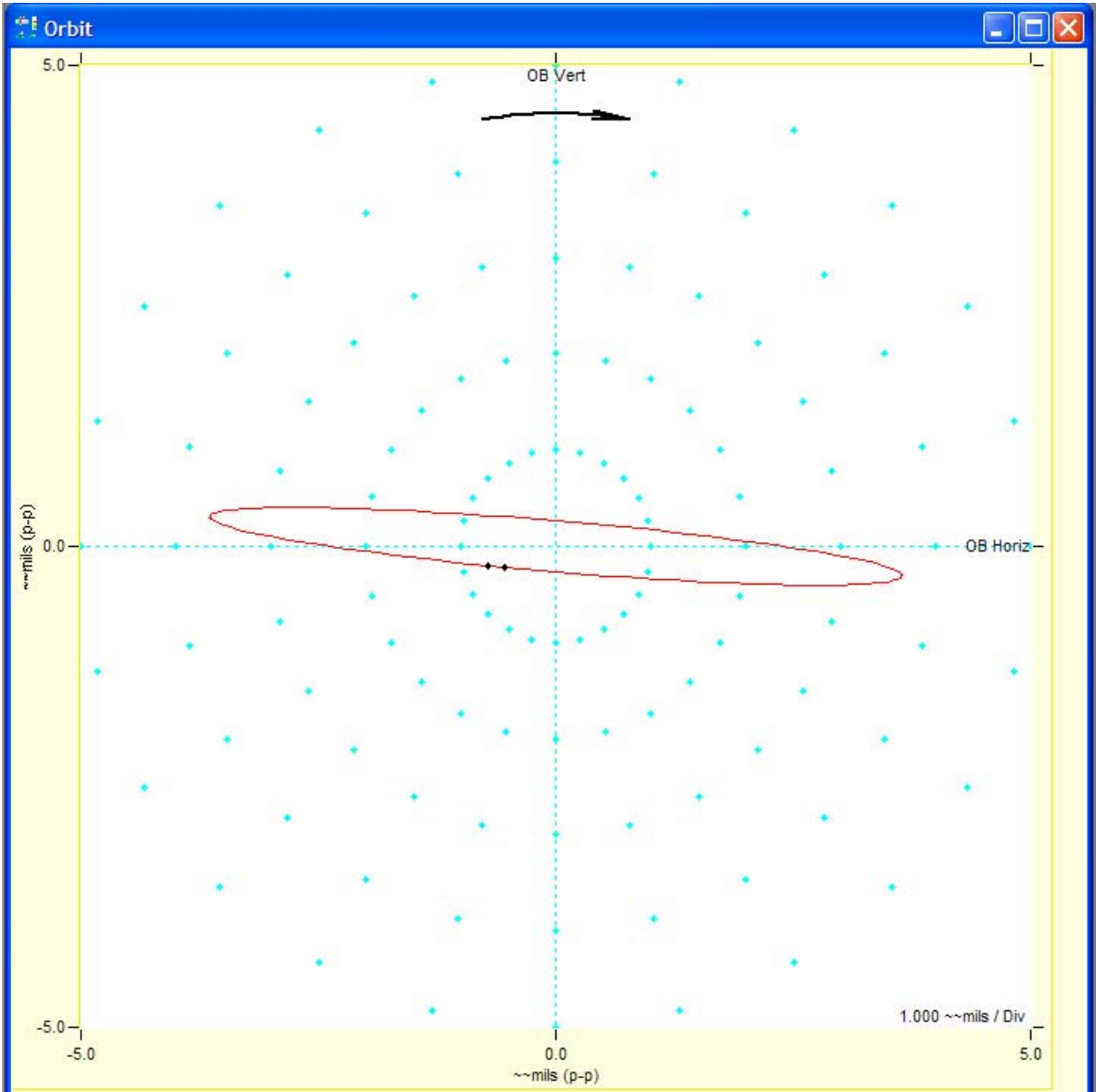


Figure 2 - Orbit for Unbalance

Figure 3 - Orbit for Casing Sensors for Horizontally Flexible Machine



Definition of Terms

Prior to getting into the balance details, it would be prudent to discuss some common terms related to balancing.

- “Field” Balancing – Most rotating components are balanced during the manufacturing process to include balance corrections of individual components (hubs, impellers, etc.) and to make corrections to assemblies of parts (rotors). Corrections made in a manufacturer’s shop will normally be done using a balance machine where the part is either mounted on a shop mandrel or else the entire rotor is balanced. In contrast, field balancing involves using vibration measurements on fully assembled machines that are usually in their final service location, and adding field corrections weights to improve the machine vibration at bearing housings or other locations.
- Single Plane Balance – This includes a balance process where corrections are made in a single axial location or at multiple locations using equal amplitude weights at the same phase (also called a “static” correction) that can successfully reduce vibration to acceptable levels.
- Multi-plane Balance – This includes a balance process where it is necessary to make balance corrections at 2 or more axial locations along the shaft where each plane will have normally different weights and phase angles.
- Vibration Vector – Vibration amplitudes for the balancing is documented as amplitude and phase. The amplitude is the magnitude of the 1xRPM vibration that is filtered using some electronic or software method. The corresponding phase is the phase angle lag for 1xRPM vibration with the lag defined as the lag angle between the firing (trigger) of the tachometer reference and the peak of the 1xRPM vibration amplitude. Any vibration amplitude unit can be used although considerations for phase difference between acceleration, velocity and displacement should be considered when locating trial weights. Vibration vectors are most commonly displayed using a polar graph to allow simultaneous display of both magnitude and phase.
- Balance Coefficient – The balance sensitivity for a given sensor to a known balance correction weight. Units are weight/response with units of grams/mil, ounces/mil, etc. and includes a phase angle that is the trial weight angle location + 180° - the response vector angle.
- Effect – This is a term used to describe the ratio between the amount of weight added and the vector response of the vibration and can be the same as the balance coefficient. Units of Weight/Response or Response/Weight are used. There appears to be no consistent definition for the effect, but units of Lb/mil, gram/mil, gram/(in/sec), mils/Lb, etc. are common. In determining the effect, the weight is the amount of added or removed weight (weight change) and the response is the vector change (tip to tip on a polar plot). The use of the term “effect” is generally limited to single plane balancing. Multi-plane balancing is generally described using influence coefficients instead of effects.

- Influence Coefficients – The ratio of vector change in vibration for a given sensor and the amount of weight added to a specific balance correction plane. Influence coefficients are best considered in a matrix form with rows and columns corresponding to sensor locations and balance correction planes. These terms are normally calculated using the multi-plane balance method described in this paper. When calculated under conditions where only a single balance correction plane is used, the results are identical to the balance coefficients for each sensor that can be obtained using a graphical method.
- Balance Lag Angle – This term is used in balancing to define the lag or delay between the passing of an unbalance weight (heavy spot) past a sensor and the peak of the vibration in displacement units (high spot) indicated from the same sensor location. It is reported in degrees, with common values of 10-70° for rotors that operate below a critical speed, and 110-180° for rotors operating above a critical speed. Electrical lag (instrument lag) is not a realistic concern with modern data acquisition equipment including laser or optical tachometers and accelerometers or proximity probes. Some instrument lag will exist with velocity coil type sensors. Some additional phase lag is likely on bearing housing seismic measurements due to damping between the shaft and the housing particularly with oil film bearings. Equipment with proximity probes may demonstrate some apparent lag if there is high housing vibration since the probes measure bearing housing to shaft vibration.
- Phase Angle – This paper only considers what is referred to as stationary phase, or the phase lag angle between a stationary point on the shaft (leading edge of a piece of reflective tape or of a notch in the shaft) and the peak of the vibration signal. Phase angles change between acceleration, velocity and displacement by 90° each. This phase angle difference must be accounted for when using the phase angle to determine the locations of trial weights.
- Residual Unbalance – This is the amount of unbalance that remains in the rotating assembly after adequate balance corrections have been made. This can be estimated using the balance coefficients or influence coefficients and would be equal to a calculated “trim” correction weight determined from the final vibration levels after balancing is complete.
- Static Balance Correction – This is a description of a balance correction made to a rotor using two balance correction weights of equal amplitude that are located on opposite ends of a rotor or impeller. This weight mounting method is commonly used for rotors that are reasonably symmetric and that may have similar vibration on each bearing that is in phase. The amount of weight added may be referred to as the total weight (2 times the weight of a single weight) or as the amount of a single weight. No standard method is currently defined.
- Couple Balance Correction – Also referred to as a dynamic balance correction. This is similar to the static correction except that the identical weight is located 180° apart from end to end. The amount of weight would either be referred to as the total weight (2 times a single weight) or the weight of a single weight, and the phase angle is usually documented as the phase angle for the weight on one end of the rotor (normally the drive end). No standard exists for documenting the angle (which weight to use) or for the amount of weight (1 or two times the actual correction weight).

- **Correction Weight** – The amount of weight added to or removed from a rotor at a specific angular location. Added weights are normally documented as positive, and removed weight as negative weight amplitudes at the removed angle. A removal of weight at one location is identical in function to adding the same amount of weight to the opposite side of the rotor.
- **Trial Weight** – This is a temporary correction weight added (or removed) for the purpose of characterizing the response of the rotor. Trial weights can be risky to add particularly for rotors that are highly sensitive to correction weight. Estimation methods have been presented in literature suggesting trial weight amplitudes that will produce dynamic forces that is less than or equal to 10% of the rotor static weight.
- **Shot or Balance Shot** – A term used to describe the process of adding trial final correction weights followed by operation to determine the vibration response.
- **Eccentricity** – The radial distance of the unbalance or the balance correction weight relative to the rotor center of rotation (radius to the correction weight). In some cases, a rotor eccentricity (instead of balance weight eccentricity) is described which is determined by the level of residual unbalance divided by the rotor weight. Rotor eccentricity is the amount of radial offset that the entire rotor would have to be eccentric by to produce the same unbalance force that would be generated by a specific correction weight mounted at a specific radius. Some standards, particularly ISO 1940, use rotor eccentricity to determine the allowable residual unbalance.
- **Projected Vibration** – Calculated vibration response that is expected after a shot is added to the rotor.
- **Trim Balance** – A balance process often called a “one shot” when previous balance data (balance coefficients) are used to calculate a shot without adding trial weights. This method is also used after a final correction weight is added to determine a smaller trim weight that can be added without disturbing the final weights that is expected to further reduce the vibration amplitudes.

Units of Measure

Field balancing can be accomplished using any vibration measurement unit that is proportional to the unbalance force. Common units of measure will include displacement or velocity although there is no technical reason why acceleration could not be used as well. For many users, displacement measurements will result in more logical positioning of trial weights since the displacement phase always identifies the “high spot”.

There may be some cases where a unit other than displacement would be preferred. Obviously if the balance specification requires verification of a velocity reading, it would be more logical to balance using velocity measurements even though the single frequency vibration can be easily converted between unit types. One particular case where using displacement may not be desired is when the test instrumentation applies some integration filters to prevent integration noise known as a “ski slope” near the left side of the FFT plot. These filters can produce amplitude and phase errors with some instruments particularly below about 10 Hz (600 RPM).

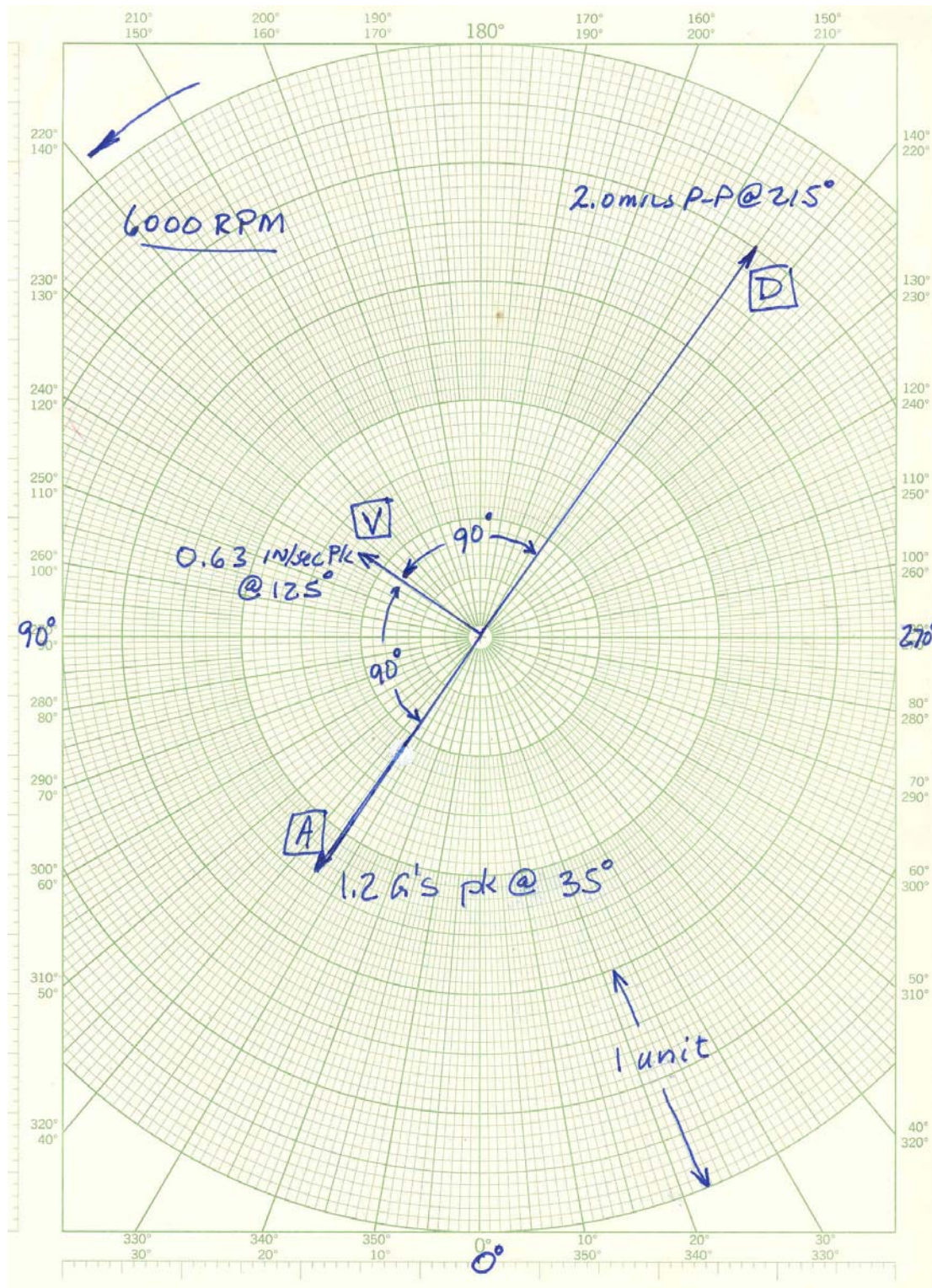


Figure 4 - Polar Plot Using Different Units of Measure for Identical Data

One very significant point to consider is the phase difference between acceleration, velocity and displacement. Any time a signal is integrated, the phase shifts by 90°. This is demonstrated on the polar plot in figure 4 as well as time waveform plots for the same data in different units in figure 5. The phase

direction change will show that acceleration leads velocity by 90° , velocity leads displacement by 90° , and due to double integration acceleration leads displacement by 180° . The test instrument used should properly display the phase angles to be relative to the unit of vibration amplitude so that the phase lag shown is the lag angle between the tachometer pulse and the peak of the filtered vibration waveform when displayed in the unit of measure.

When using the vibration phase to determine and angular position of trial weights, the phase measurement is normally converted to a displacement phase (the actual “high spot”) with weights located relative to that phase with some consideration for the lag angle between the weight (“heavy spot”) and the peak phase (“high spot”).

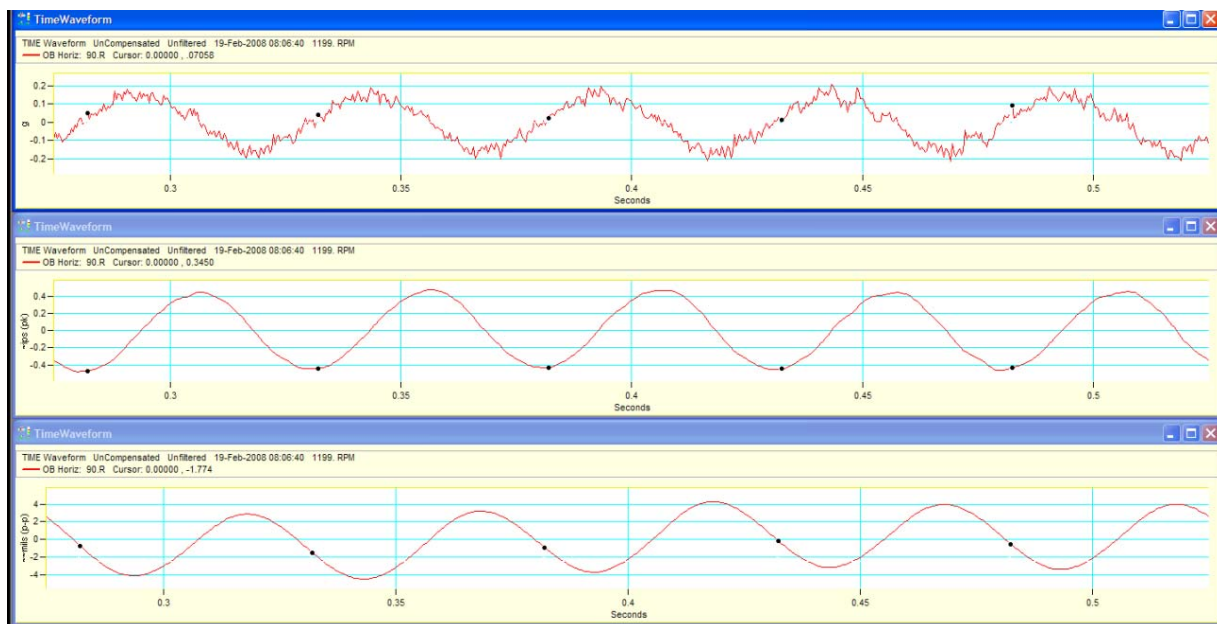


Figure 5 - Time Waveform for Different Units of Measure

Balancing Assumptions

There are some basic assumptions that are made when doing field balancing. These include: linear response, accurate/repeatable test measurements, and consistent weight placement. These sound like simple assumptions, but these can produce significant problems during a particular field balance.

Linear response of the system simply states that if I get a response of 1 mil for a certain weight size that I should get a 2 mil response for twice that amount of added weight. This will normally be mostly true unless the rotor is severely out of balance. In general, vibration in excess of about 0.3 in/sec peak will begin to produce non-linearity so that it will take more (or less) weight to produce the same amount of vibration change. This nonlinear reaction is the primary cause for overshooting (driving the response 180° past the center of a polar plot) when a final correction weight is added if the reference vibration was high ($\gg 0.3$ in/sec). In this scenario, the trial weight will characterize the response in the nonlinear range producing a calculated weight that is excessive or that is moderately off in phase angle. In some

cases, the phase of the balance coefficients can change with vibration amplitude making weight placement errors as high as 20-40°.

Accurate and repeatable vibration amplitudes sound obvious. Currently available test instruments will produce consistent amplitude and phase readings for the same input signals in most cases. However, the accuracy of the readings can be affected by the placement or movement of the vibration sensors or movement of the tachometer (phase) reference sensor. So if somebody moves a sensor between runs, or the tachometer is pointing to the reflective tape but 10° away from the previous run, you will not get dependable balance calculations. Variable readings can be even more difficult to identify, and can be caused by thermally induced rotor bows, loading differences, process temperatures, alignment offsets due to variations in casing temperatures, etc. Therefore, it is absolutely mandatory to assure that the same machine conditions are used during vibration measurement for each balance run including rotor speed, machine load, heat levels, etc. Being patient and acquiring consistent data will generally produce quicker results on most balance jobs than rushing to get finished at the cost of less accurate data.

Placement of trial weights and final correction weights can be inaccurate if a care is not used in weight placement. Determining the actual location of the tachometer firing (normally the leading edge of the reflective tape) relative to a position on the rotor where the correction weights are added can produce large phase errors. To eliminate this risk, once a balance process is started, the rotor should be clearly marked with angular positions so that all additional weight additions are properly made relative to the assumed phase angle for the initial trial weights. If the weight additions are done in this fashion (particularly if there is not previous balance data and you aren't trying to reuse the balance response in the future), the possible phase error between the rotor weights and the actual tachometer position is not significant since the balance corrections are all made relative to the trial weights.

Balance Tolerances

Field balancing tolerances are not well defined in available literature except in some requirements defined by end users or equipment manufacturers. Many specifications are available that define allowable residual unbalance levels for a shop balance such as ISO 1940, ANSI S2.19, and API. However, these specifications generally do not specifically define a level of vibration or residual unbalance in an assembled machine at operating speed. When field balancing equipment, the following tolerances are suggested as reasonable targets for some machine types using 1xRPM filtered vibration amplitudes:

- Turbomachinery (API compressors, turbines, etc.) – Mils Pk-Pk < $\sqrt{(12,000/RPM)}$ using shaft to bearing housing relative vibration
- Large sleeve bearing equipment without proximity probes – < 0.05 in/sec pk on bearing housings
- Antifriction bearing centrifugal fans/blowers – < 0.1 in/sec pk on bearing housings
- Antifriction bearing motors – < 0.08 in/sec peak on the bearing housing or motor frame

In general, field balancing is ideally done until the phase readings become unstable due to the low amplitudes of vibration. However, practical field balancing is frequently finished based on a limited number of balance shots, by time or on the capability and insistence of the balancer.

It should be noted that for cases where very high levels of vibration are observed in the initial reference run, the nonlinear response may require starting the balance process over once the vibration is at reasonable levels.

Weight Corrections

Balance weight corrections can be done in a number of ways. When performing a field balance, it is generally desirable to have some method of making trial weight moves that can be easily mounted and removed once final weights are determined. Some trial weights can include:

- Clamp on balance weights (these are commercially available in a wide variety of weights, shapes, installation method and material type)
- Balancing putty (the same stuff used on a shop balance machine)
- Added bolts/washers/nuts
- Engineered weights (dovetail slots, balance plugs, etc.) on machines that have removable balance weights such as power turbines and generators

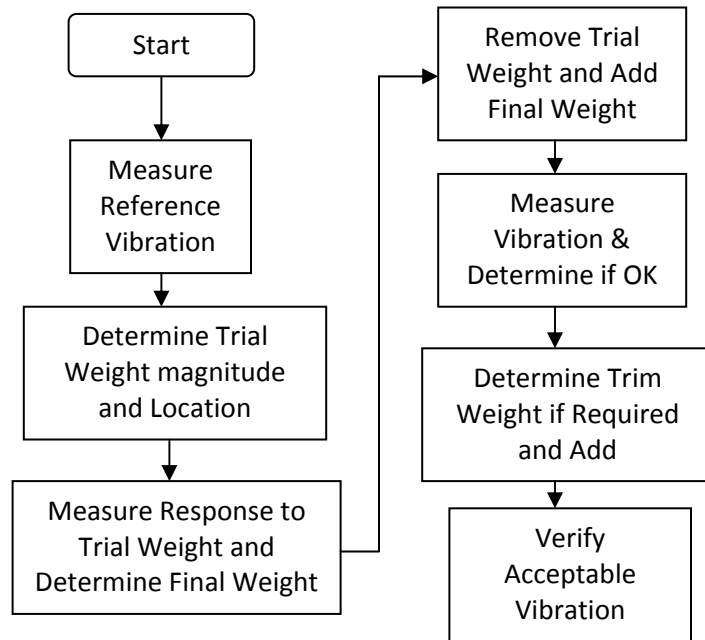
Final correction weights are by definition intended to be left permanently on the rotor to correct for unbalance. The final correction weights will include:

- Welded plates
- Engineered weights (same as above)
- Clamp on weights (when welding is not practical)
- Removing weight (drilling holes, grinding, etc.)
- Bolted on weights (added bolts, washers, nuts, etc.)

I would always prefer to inspect the rotor for causes of unbalance to determine if there is a good reason for the rotor being out of balance and to help identify other problems that could exist. Particularly on fans, the presence of a lot of fouling may warrant cleaning it off instead of field balancing. In addition, cracks or damage to impellers should normally be repaired prior to making balance corrections. In some cases, previous balance weights may have come off due to being poorly installed or improper materials (corrosion). The other benefit of this type of visual inspection is the benefit of looking at previous balance weights that have been used as a mental reference point for the selection of trial weight magnitudes. Please review the trial weight selection section for additional guidance in selecting trial weights.

Single plane balancing is the process of making balance corrections to a rotor in only one axial location. Single plane balancing can be very useful even for rotors that would normally be corrected using a two plane balance method when vibration amplitudes and phase readings allow static or couple corrections alone. The process of single plane field balancing is shown below.

It is also common after addition of the final weight (CW) to need to add a smaller trim weight that is added to further reduce the vibration while leaving the final weight in place.



The general way that single plane balancing is completed is by using a graphical method as shown in figure 6. Numerous published works describe how to perform single plane balancing using graphical methods. As an alternative to the graphical method, a computer program can also be used for single plane balancing. Based on the huge benefit of an intuitive approach to single and multi-plane field balancing, the user is strongly advised to become proficient with a single plane graphical field balancing method prior to advancing to computer based solutions.

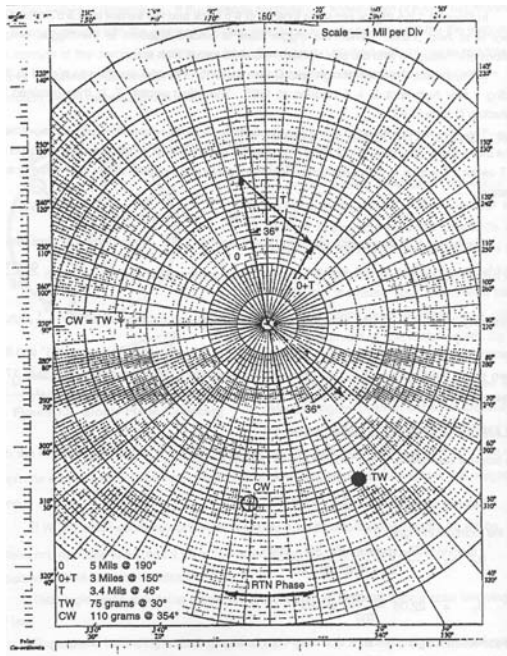


Figure 6 - Graphical Balance Example

Computer based balancing routines will calculate solutions based on the input data. When the data is accurate and the system is linear, the response to balance corrections can be accurately calculated using the analytical method described in the following sections. However, fluent use of graphical tools can provide huge benefit in reviewing balance responses particularly when a balance job has not gone well.

The methods described above, particularly the graphical method, are ideal when balance corrections are made to a simple rotor using a single vibration measurement location. Most practical field balancing will involve measurement at multiple locations on the machine. Balancing using a single measurement point should be capable of minimizing the vibration at that point. However, use of a single measurement location will usually result in very low vibration at that location and higher vibration at another point. This difference is generally the result of the unbalance correction not being added at the actual point of unbalance on the rotor, or unbalance in multiple planes. When this occurs, and a single plane balance correction is desired, it is necessary to use multiple measurement points and minimize the vibration at several points at the same time. The use of multiple points will usually get more complicated than is easily resolved using the graphical method.

Multiple Point Single Plane Balance

The use of multiple measurement points now requires an analytical approach to best minimize the vibration at all points simultaneously. The graphical technique can be used, but it is common that different points (i.e. vertical vs. horizontal) will have different reaction to a balance weight. Since the different points will have different effects, it will be necessary either to graphically review the response of each point, or to use a numerical method.

The recommended method used to calculate balance correction weights (“shots”) when multiple sensors are used is the least squares numerical method. This method uses a numerical procedure to reduce the sum of the squares of the amplitudes on each measurement point, and is the most common method used in balance programs. With this method, the highest amplitudes are more heavily “weighted” to help drive the general vibration severity down.

Before the numerical process can be described, the concept of vibration and weight changes must be well understood. With a single plane method using one sensor, the original vibration is measured (O), a trial weight is added (TW), and the response with the trial weight is measured (O+T). The original vibration run will normally have no trial weight, so the weight change equals the trial weight. The vibration *change* is called the trial response (T), and is found by determining the amplitude change from O to O+T from a vector plot.

When this is shifted over to a numerical process, terms are defined for each of the values described above but by using subscripts to help accommodate conversion to a computer program:

- V_{ij} = Vibration vector (amplitude and phase) measured at point i during run j
- Point i – Measurement points will be used with $i = 1, 2, 3, \dots$ based on the number of measurement points used (no limit to the number of points)

- Run j – Runs include the reference run (Run 1), as well as the trial run (Run 2), followed by any additional sets of vibration data (Run 3, Run 4,)
- W_j = Weight vector (amplitude and phase) installed when Run j was recorded
 - Note that W_1 should normally be zero (no weights added on initial reference)
 - Weight addition can become a complicated accounting process with resulting confusion between trial/final weights added as well as trim weights. Recommendation is to document weights using clear notes regarding weights that have been added and removed throughout the process to prevent confusion.
- ΔV_{ijk} = The vibration vector change between Runs j and k at point i
- ΔW_{jk} = The weight vector change between Runs j and k

Using these definitions, the balance calculations are as follows for use of a single vibration sensor and a single correction plane (refer to figure 6):

Initial weight – $W_1=0$ (no initial weight)

Initial vibration – V_{11} = Original amplitude and phase (“O”) at point 1 for run 1

Trial weight – W_2 = Trial weight magnitude and phase (“TW”) is the weight installed for run 2

Trial run vibration – V_{12} = Vibration with added weight (“O+T”) at point 1 for run 2

$\Delta W_{12} = W_2 - W_1 = W_1$ (“TW”) is the weight change from run 1 to run 2

$\Delta V_{112} = V_{12} - V_{11}$ (“T”) is the vibration change from run 1 to run 2 at point 1

Effect = $\Delta V_{112}/\Delta W_{12} = \text{“T”/“TW”}$ is the influence coefficient for point 1

Final Weight = FW (remove trial and add final weight)

$$\text{Calculated final weight} = \frac{V_{11}}{(\Delta V_{112}/\Delta W_{12})} = \frac{(V_{11} \times \Delta W_{12})}{\Delta V_{112}} = \frac{O \times TW}{T} = FW$$

Using this description, $\left(\frac{\Delta W_{12}}{\Delta V_{112}}\right)$ is the balance coefficient and $\left(\frac{\Delta V_{112}}{\Delta W_{12}}\right)$ is the influence coefficient for sensor 1 due to the trial run in plane 1 (the only correction plane used). Please note that each value shown above is a vector quantity that includes amplitude and phase.

Show a similar plot but with two weights (trial and final) and show the effect calculation for each run 0-1, 0-2, 1-2

This must certainly sound like a seriously overcomplicated way to describe a simple balance calculation, but this added complexity is required for using multiple points and/or multiple planes. The other item excluded from the description above is the determination of the phase angle for the final weight. Using the graphical method, the angle between T and O is used to determine the amount of shift required for the balance weight. The numerical method described tracks the phase positioning by using the combined amplitude and phase by using complex math variables, where a single complex variable contains both amplitude and phase data (actually real and imaginary or quadrature response). Therefore, with the numerical method, the resulting final weight as shown above would include both amplitude and phase definition.

If the same process is repeated using multiple sensors using the graphical method or the calculation procedure described above, different final weight amounts and positions will be calculated for each sensor. The variation in the calculated weights will be due to variations in the data accuracy or more likely due to the vibration at the sensor location not being the result of unbalance at the correction plane. For sensors where low sensitivity exists, the calculated balance correction weight using that sensor may be very large (or small) and unrealistic. The example below for a large motor balance demonstrates this well:

Example – large motor with rotor unbalance. A single plane trial weight (static correction with identical weights added in phase on opposite ends) was used, with the final weight calculation done using single point method for each sensor individually with the data shown in table 1. The same data was then used to calculate a combined correction weight for all the points. This combined weight is not the average of all the calculated individual weights, but rather a least squares optimization that will best minimize the general vibration for all sensors simultaneously.

Table 1 - Multiple Point Balance Calculation Data

Sensor Description	Reference Vibration (no weights)	Trial Run Vibration (added 170 gm @ 270°)	Calculated Weight using individual points
Outboard Horizontal	6.55 @ 214	4.70 @ 219	585.7 @ 282°
Outboard Vertical	1.05 @ 99	0.60 @ 125	312.5 @ 297°
Inboard Horizontal	4.90 @ 215	3.50 @ 219	586.1 @ 280°
Inboard Vertical	0.60 @ 103	0.44 @ 113	559.5 @ 295°
Combined weight		Using all sensors->	570 @ 282°

When multiple sensors are used, the influence coefficient for each sensor can be determined by calculating the vector vibration change and the change in the trial weight as shown above with the results configured into an influence coefficient matrix as shown below:

$$|C| = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{bmatrix} \text{ where } C_i = \left(\frac{\Delta V_{i,2}}{\Delta W_{1,2}} \right) = \frac{\text{Vibration Change for Sensor } i}{\text{Trial weight change}}$$

The reference vibration vectors would also be arranged into a similar column matrix as:

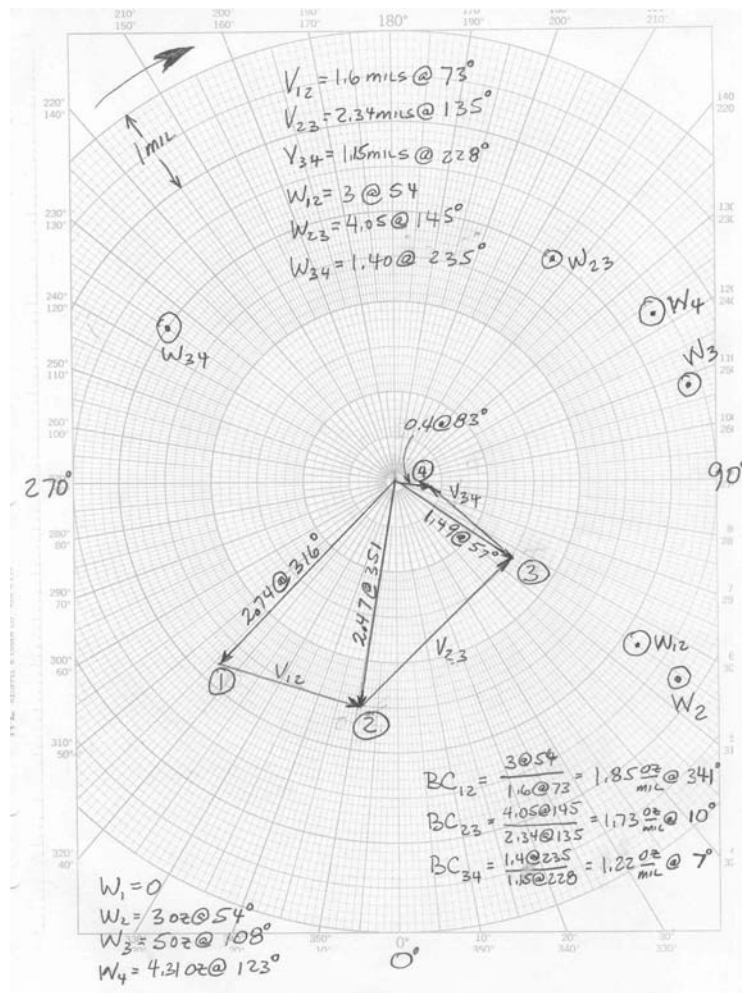
$$|V| = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} \text{ where } V_i \text{ is the reference vibration amplitude/phase}$$

The final correction weight will be calculated relative to this reference vibration matrix. The final balance correction weights are calculated using the following matrix operation:

$$M = -[C^T C]^{-1} \times C^T V$$

C^T is the conjugate transpose of C and is found by transposing the matrix and changing the sign on the imaginary component of each element. The solution final weight M will produce the lowest combined vibration by minimizing the squares of each sensor value.

Once the process is converted to a computer program that can handle the math, the equations will support calculating a new set of balance coefficients (actually influence coefficients) every time a weight is added to the shaft. This is done by using the vibration change and the weight change for each run. Actually if two shots are completed, three sets of balance coefficients can be calculated including Run 1-Run 2, Run 2-Run 3, and Run 1-Run 3. This concept can provide some tremendous help particularly if there is a lot of non-linearity in the data, or if for some reason you have one set of "bad" data for a given run.



To reduce the errors and variations as much as possible, it is generally more accurate to base future calculations on the most recent runs since vibration levels are generally decreasing as weights are added. This is similar to the process of "taking a new O" as has been taught for years using the graphical method. However, if capable software is used that can handle the reference selection, the software can be allowed to "take a new O" on every run through the process. This is accommodated since the balance coefficients are determined from vibration and weight changes. The software is, in concept, capable of dealing with calculation of these differences provided that each set of vibration data is documented with a corresponding set of weights. A graphical description of this process is shown in figure 7, which shows the non-linearity in balance coefficients as the vibration amplitude is reduced.

Figure 7 - Graphical Example of "Taking a New O" on Each Shot

The example in figure 7 shows a classical example of balance coefficient change with vibration amplitude. The initial trial run was used to calculate a correction weight of about 5 oz at 108°. Although

this is properly calculated for the reference and trial run, the results of that balance shot were much less than desired. Review of the balance coefficients as the machine was getting smoother indicates that the initial balance coefficients used for the first two shots were off by about 30° compared to those determined from the final two runs. If a “new O” would have been used numerically (use BC₂₃ to calculate a trim correction from run 3), the last shot would have produced significantly better results.

Projecting Results

One of the better methods for improving the balancing process is by using either manual or computer methods to predict or project the vibration that should result after a set of calculated weights are determined. Using this approach, the projected vibration values should be approaching zero when using multiple measurement points.

If the calculations include a single measurement and a single balance correction plane, the calculated balance correction weight should produce zero vibration at that specific measurement point. Once multiple sensors are used, they will usually be minimized together as described above. When that is done, some points will reduce better than others, but there is no easy way to tell without projecting results from the balance data. Projected vibration results can be determined by using the influence coefficients, the reference vibration data, and the calculated correction weight as follows:

$$|V_{\text{projected}}| = |V_{\text{ref}}| + |C| \times M$$

Review of the projected vibration values will provide tremendous insight on the quality of the data and the linearity of the balance process. If the projected vibration is at or near the same amplitude as the reference vibration, then the balance corrections are either not near the actual unbalance, there is inconsistency or non-linearity in the data, or else the vibration is not the result of unbalance. Continuing to add balance correction weights in an attempt to further reduce vibration once projected vibration amplitudes are not decreasing (along with the actual vibration after adding weights) is not advised.

Example of projected vibration – the previous example from table 1 is used to calculate the projected vibration after addition of the 570 gram weight at 282°:

Table 2 - Projected Vibration on Motor Balance

Sensor Description	Reference Vibration (no weights)	Projected Vibration (added 570 gm @ 282°)
Outboard Horizontal	6.55 @ 214	0.18 @ 227
Outboard Vertical	1.05 @ 99	0.94 @ 247
Inboard Horizontal	4.90 @ 215	0.23 @ 163
Inboard Vertical	0.60 @ 103	0.13 @ 192

The projected vibration results above show that the vibration on the rougher points should be well reduced, but for some reason the outboard vertical appears not to be responsive to correction weight. It should be noted that the calculated balance correction weight using only that sensor also showed a

significantly different correction weight than the others. This would suggest that this point is not a good sensor location to use for the balance calculations.

Selecting a Trial Weight

Selecting a trial weight can be a dangerous proposition. In many cases, the vibration on the machine is already high, and adding an excessive trial weight could increase the vibration amplitudes. If the vibration is already high, you normally don't want to make a lot of trial runs without reducing the vibration. So it would be very nice to at least get in the correct quadrant for the trial weight location and add enough weight to get some response. In all cases, the ideal trial weight would be one that provides enough effect to calculate a final weight from, yet small enough that, if the phase lag is much different than expected, the vibration won't significantly increase. This selection is not easy unless a reasonable amount of information is known about the machine. Adequate trial weight would be considered to be enough weight to produce a 10% change in vibration vector based on the original vibration. For instance if the original vibration is 2 mils, the trial response should be a minimum of 0.2 mils change on a polar plot. Accuracy of the calculations will certainly be better with higher response provided that the vibration amplitude is not increased (possibly producing non-linearity).

Trial weight selection can be done in several ways, but is always dependant on the weight of the rotating assembly and the speed of the shaft. Jackson has suggested use of a trial weight that will produce a dynamic force that equals 10% of the static weight or less,

$$TW = 56,333 \times \frac{\text{Rotor Weight}}{RPM^2} \text{ Oz} - \text{In}$$

If the balance is to be a static correction (equal weight in phase on opposite ends of the rotor), the trial weight above should be divided into two equal weights with one of the half weights added to each end of the rotor.

Other trial weight estimating methods include the API residual unbalance limit of $4 \frac{\text{Weight}}{RPM} \text{ Oz} - \text{In}$ and ISO 1940. ISO 1940 limits are specified with a range of G limits with unbalance levels depending on weight and speed as follows:

$$TW = 6.015G \frac{\text{Rotor Weight, Lb}}{RPM} \text{ Oz} - \text{In}$$

Common G levels include the API limit (identical to G0.67), and G2.5 (common for turbines and large motors) and G6.3 (typical for fans and pumps). The ISO 1940 has a range of G values that correspond to different machine classes. Some of these are overlaid in figure 8 for a 1000 Lb rotor.

As observed in figure 8, the 10% journal reaction method produces the highest trial weight amplitude for low shaft speed. It then becomes conservative at higher shaft speeds. In general, higher speed machinery will usually have lower ISO 1940 G levels specified. In contrast, lower speed and less

precision equipment will often have much higher specified G levels. So in the final conclusion, Jackson's recommendation of the 0.1G limit may be the most appropriate choice.

If a machine has a critical speed or structural natural frequency near the operating speed, a trial weight size equivalent to the API limit (G0.67) could be excessive. In other cases, a trial weight much larger than those suggested may not be adequate to get a good effect. In general, ALWAYS use the smallest trial weight necessary to get a useful effect and use a second trial run with more trial weight if it is necessary to increase the trial weight to get an effect.

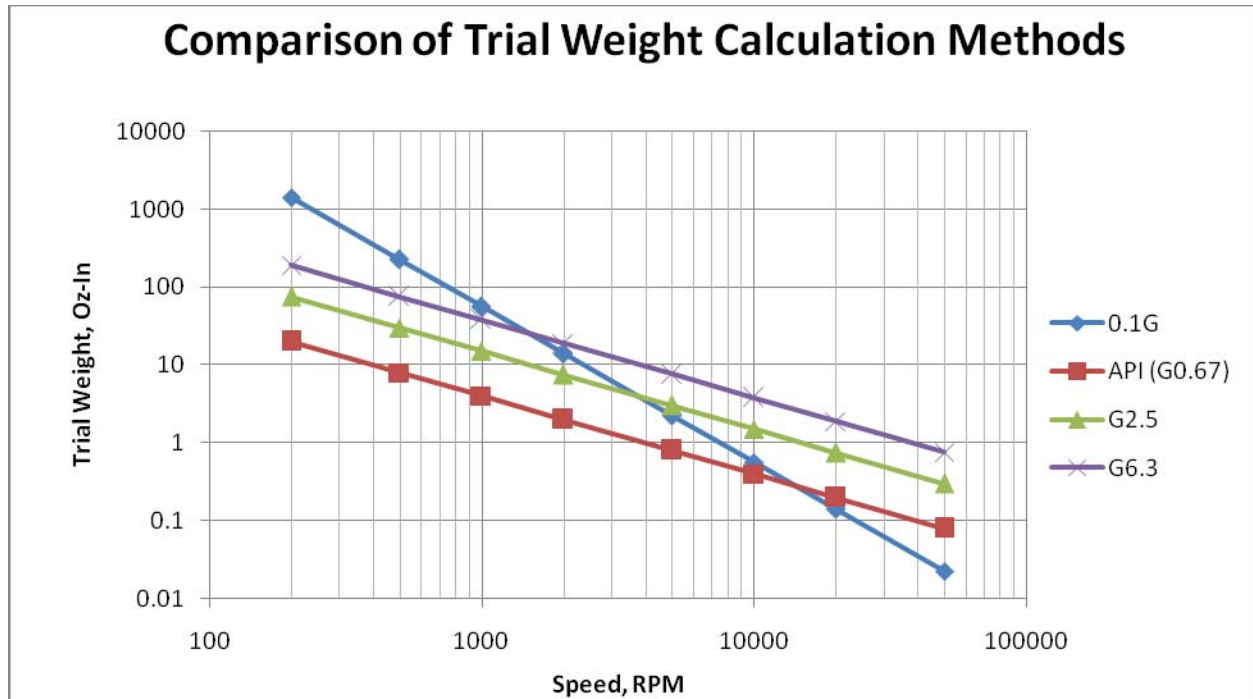


Figure 8 - Comparison of Trial Weight Calculation Methods

Locating the First Trial Weight

In addition to selecting the amount of trial weight to add, the next critical item is the phase location of the trial weight. Ideally, the trial weight should be located opposite the heavy spot (or weight removal at the heavy spot). Unless previous balance experience is known for the machine, or if you know if the machine operates above or below a critical, selecting the first trial weight location can be a guess.

There should always be a fairly repeatable phase lag on a given machine at constant speed. This lag is made up of the mechanical lag (shown in figure 9) plus any electrical lag. If there are oil film bearings or other damping elements, additional phase lag particularly on bearing housings can be present with an additional 10-60 degree lag. The lag in figure 9 assumes that the unbalance is located at 0°.

Assuming a machine operates at least 10% below the 1st critical speed (2000 RPM in figure 9), the mechanical phase lag should be less than 20° or so. The mechanical lag plot above will also suggest that phase lag angles will always be between 0 and 180°. It has been suggested by some that a “safe”

trial weight position when nothing is known about the machine is 90°! This phase lag will almost always produce a trial run that will at the worst produce similar amplitudes of vibration but appreciable phase change which is certainly adequate for balance calculations. It should be noted that in theory it is not reasonable to expect a phase lag to ever be higher than 180°.

If the rotor speed is high enough that it approaches or passes a 2nd critical speed, the phase lag will shift back to near zero and increase to 90° at the 2nd critical and then towards 180° well above the 2nd critical at the balance plane that is dominant. This principal can be used for higher speed equipment to determine which end of the machine is likely the predominant contributor since the phase lag will generally return to the same value as the rotor approaches each critical speed.

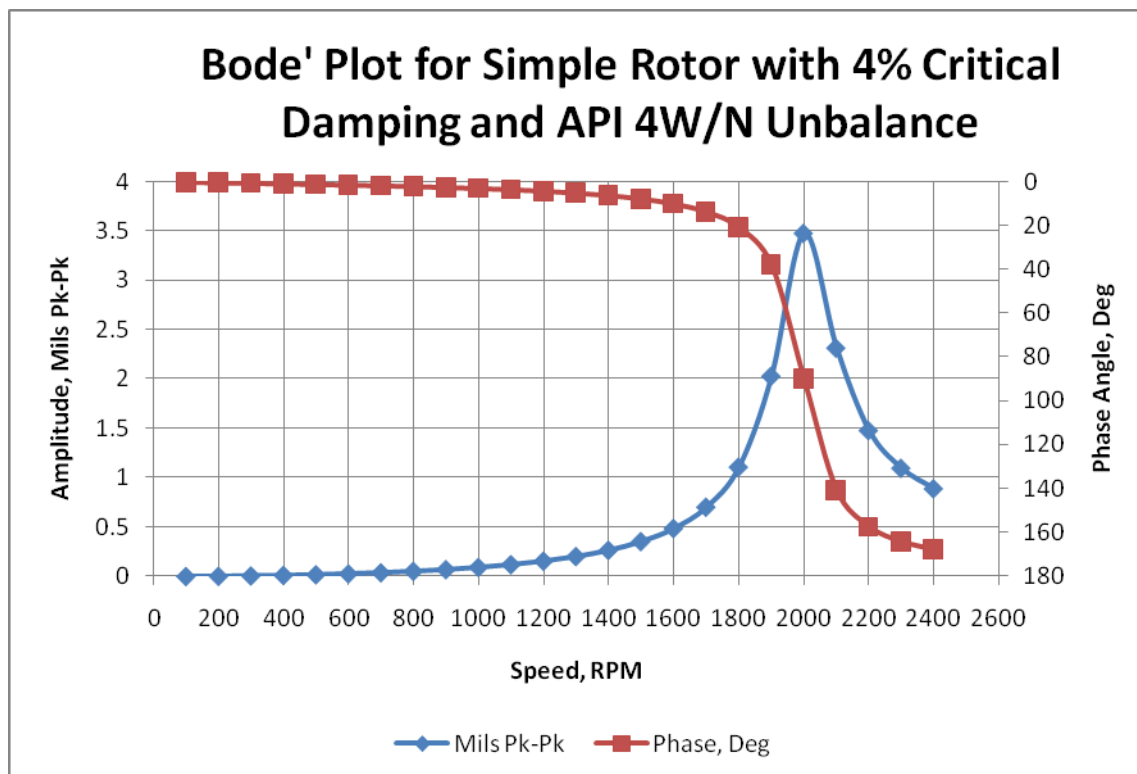


Figure 9 - Mechanical Phase Lag Due to Critical Speed

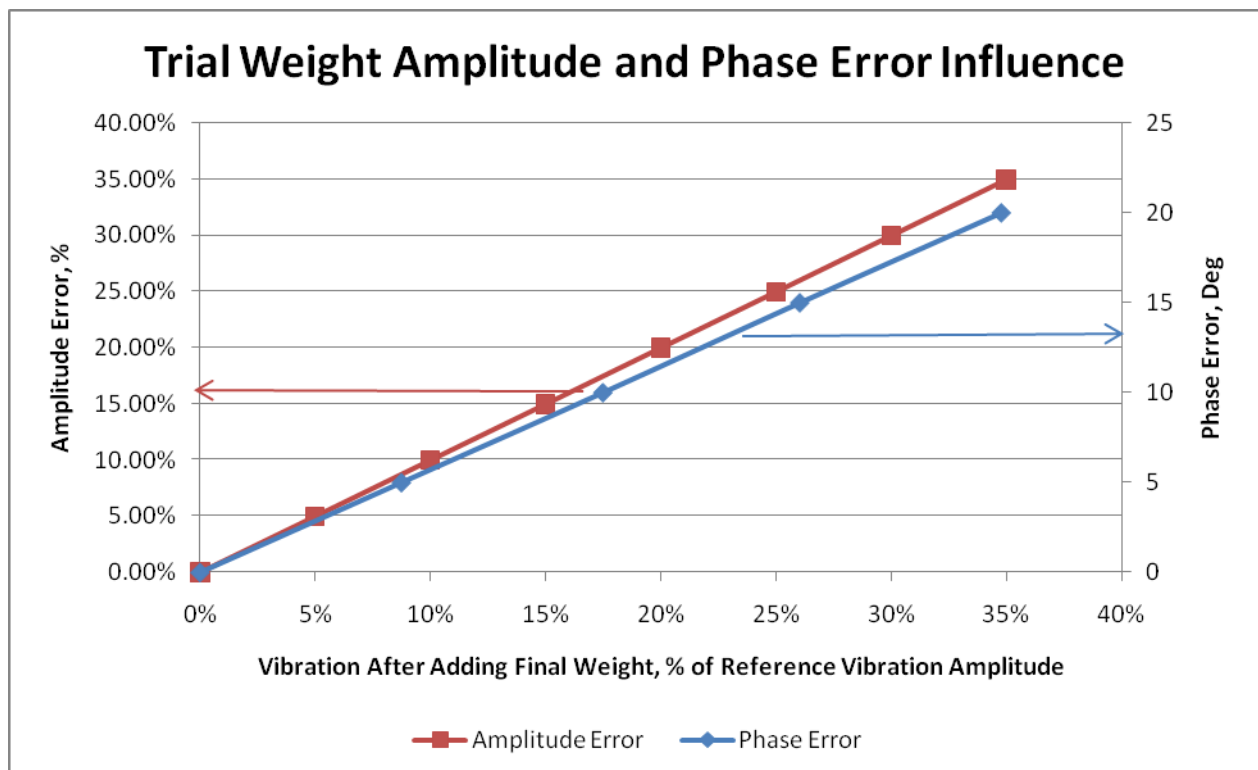
Difficulties with Single Plane Balancing

Some balance jobs just don't go well. The most common problem is likely readings and trial weight locations confused on various runs. If the process flows easily with a reference, trial and final weight, there will usually be very good results. If several trial weights are required, or if there is a lot of non-linearity (inconsistent balance coefficients), or if you are alternating between "final" and "trim" weight methods, it is very easy to make some bookkeeping errors regarding weight changes and what references to use.

Here are some common problems:

- Balancing syndrome – Viewing every vibration problem as a balance problem and continuing to balance after the balance effort is not producing acceptable results
- Sources of vibration other than unbalance – Shaft misalignment, cocked bearings, bent shafts, etc.
- Unbalance not in correction planes – If the unbalance is at a different location than where corrections can be made, the projected vibration will never decrease for all sensors
- Inaccurate data – Movements in the tachometer phase position or sensor locations, lack of data averaging, varying shaft speed, etc.
- Thermal shifts – Caused by coupling alignment changes due to thermal growth, thermally sensitive rotors, etc.
- Improper documentation – Improperly documenting weight locations during the process, or using an incorrect reference when using a trim calculation

One of the most basic of problems is inaccurate data used in the balancing process. Variation in the amount of weight is generally pretty small since weights are either weighed or known. However, variation in the angular position of trial and final weights can be large particularly if you aren't real careful when adding weights. The influence of the amount and angular position of weights is shown below indicating that the angle for weights is far more important than the actual amount of the weight.



Non-Linearity of Response

Anyone who has done much field balancing will know that balance coefficients will change if the vibration amplitudes start out very high. As a general rule of thumb, any machine with over 0.3 in/sec peak is likely to have non-linear response. Some machines will be non-linear at far lower vibration amplitudes. What this means is that using the standard methods of balancing will result in continual overshoot/undershoot with successive weight calculation as the balance coefficients change unless a method is used to recalculate the balance coefficients as the machine gets smoother. This is the concept of “taking a new O” that has been taught for years in many balance courses.

To demonstrate this concept, a rotor kit was used with a configuration as shown in figure 10. The rotor was balanced to extremely low levels in two correction planes (at the disks). Once the rotor was extremely well balanced, unbalance weights were added to the drive end disk and vibration response was recorded. The sample points documented included horizontal proximity probe and seismic (accelerometer integrated to displacement) at each bearing location. Points were labeled as follows: DES – drive end seismic, DEX – drive end proximity, NDES – non-drive end seismic, and NDEX – non-drive end proximity. The tachometer reference signal was located in phase with the vibration sensors.

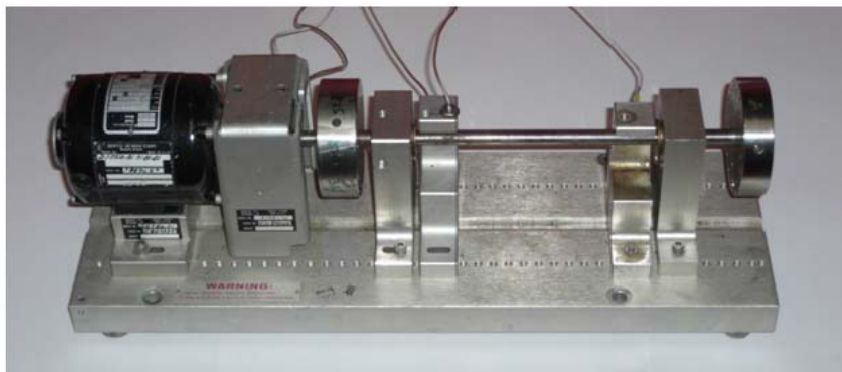
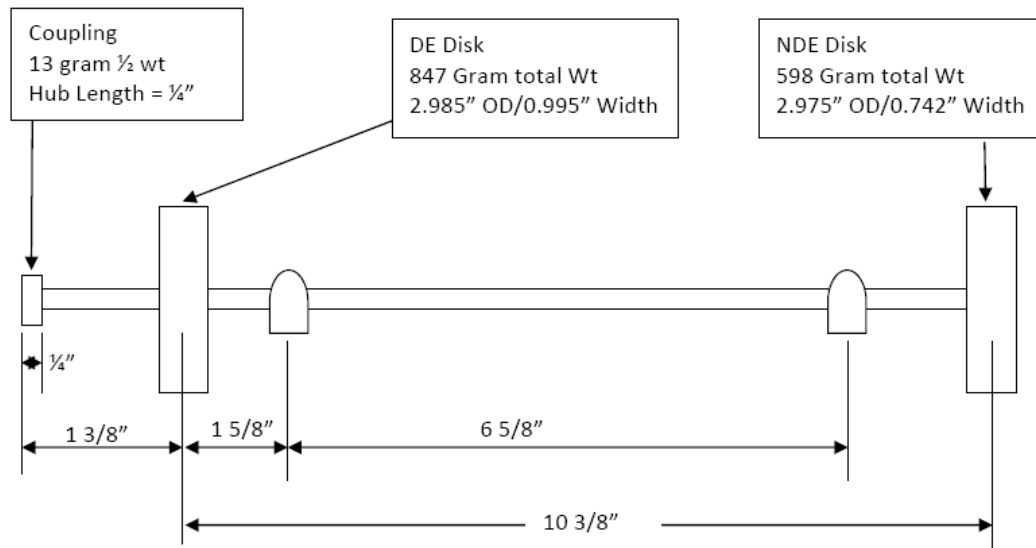


Figure 10 - Rotor Kit Layout

Influence coefficients (mils/gram) were calculated for each amount of unbalance weight using the well balanced state as the reference with proximity probe readings runout compensated. The maximum vibration amplitude measured on the proximity probes was a little over 6 mils pk-pk for the highest applied unbalance, while the seismic sensor never showed over 2 mils pk-pk. Figures 11 through 13 show the vibration response as well as the calculated influence coefficients separated into amplitude and phase plots. It is easily observed from the plots that the balance coefficients are certainly non-linear and strongly dependant on the vibration amplitude.

If the vibration response was linear, the vibration response should increase in a straight line as the unbalance weight is increased. The influence coefficients show that the balance response can vary by a factor of 1.5 to 3.0 as the vibration increases. In addition, the phase lag angles are shown to generally increase with increasing vibration, with an additional phase lag of as much as 45°! With these sorts of variations, it becomes clear why a common rotor kit is not a good demonstrator for field balancing

unless it is very well balanced from the start and only small unbalance weights are used for demonstrations.

Based on this example, a severely unbalance rotor may result in significant phase lag and large amplitude errors for calculated balance weights until the vibration is reduced to more linear ranges (see also figure 7). This is the reason why “taking a new O” is commonly required for machines that start out very rough. However, as described above, it is possible to allow the software to accommodate this if appropriately programmed to handle the “new O” by conducting the balance calculations using any two balance runs where vibration and weight locations are known without having to start over.

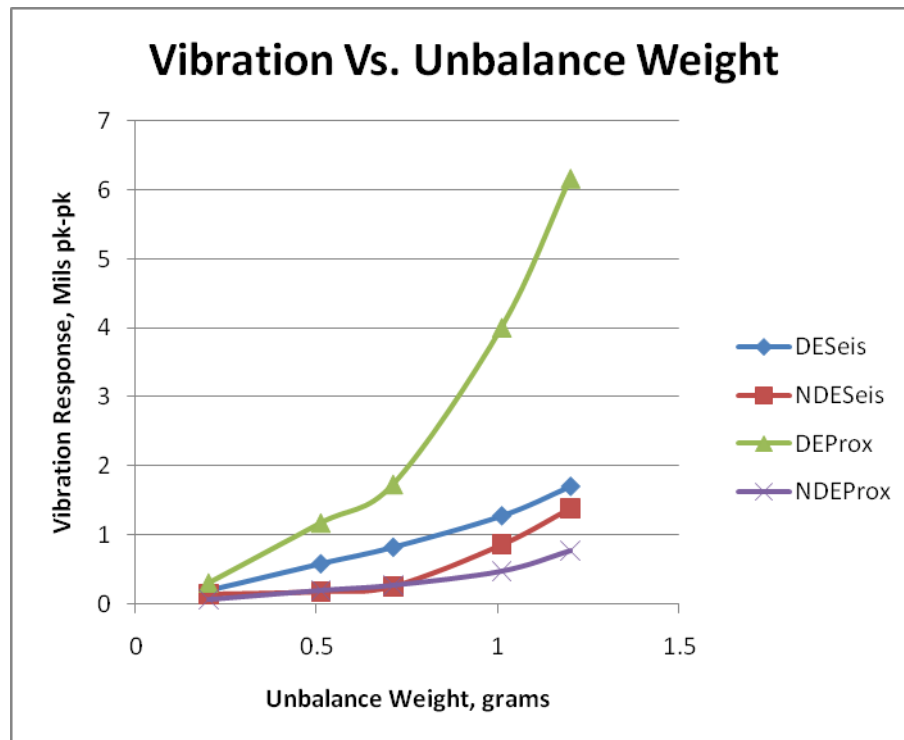


Figure 11 - Vibration Amplitude vs. Unbalance Applied

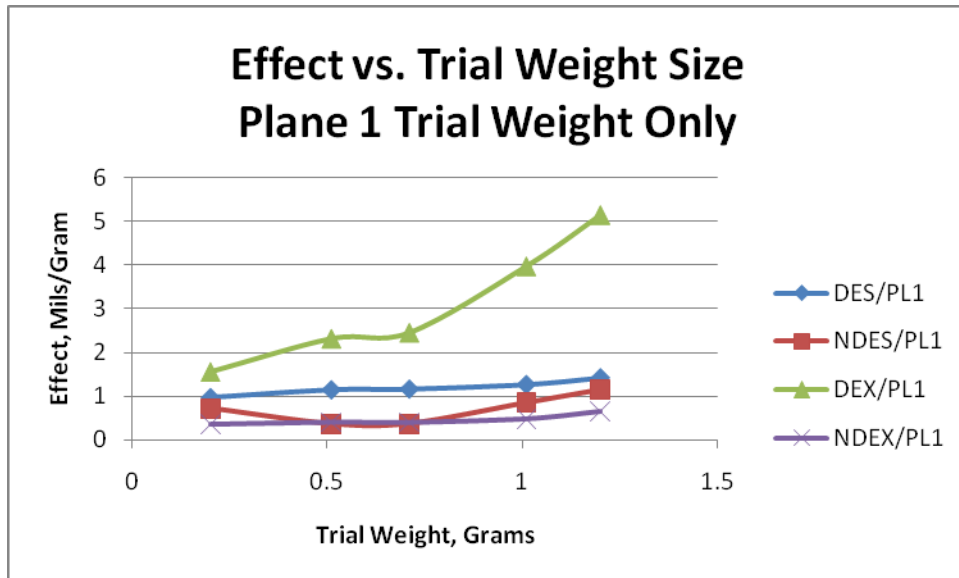


Figure 12 - Influence Coefficient Magnitude vs. Trial Weight Size

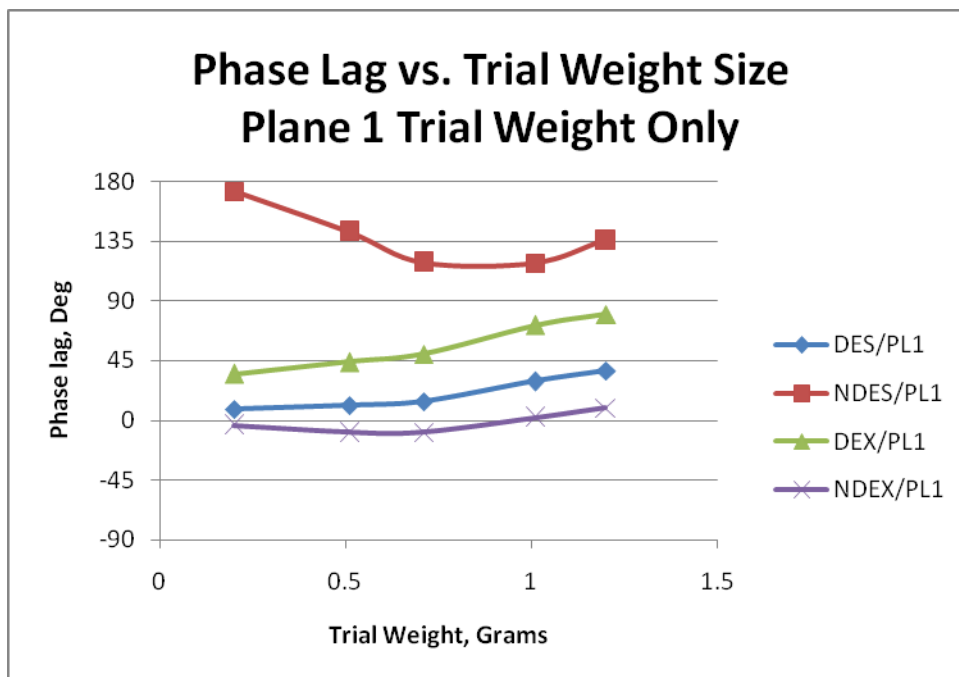


Figure 13 - Influence Coefficient Phase vs. Trial Weight Size

Multiplane Balancing

Multiplane balancing is an extension of the same methods used for single plane. The primary difference is that trial weights must be added to each correction plane to determine the balance response at each sensor to the various weight plane locations.

The simplest multiplane balancing can be accomplished using the static/couple method. This is a balancing process that can be used on reasonably symmetric rotors that requires the vibration readings on opposite ends of the rotor to be resolved into in phase (static) and out of phase (couple) readings. When the rotor is symmetric, a two plane balance problem can be separated into two single plane balances with good results.

Multiplane balancing in a more general sense involves simultaneously solving the balance problem by adding different correction weights in more than one correction plane and at different phase angles. This method is required for non-symmetric or highly flexible rotors, and can accommodate any number of correction planes, rotor speeds, and vibration sensors.

As a minimum, there must be at least as many sensors as correction planes. For instance, a two plane correction will require at least two sensors, normally on opposite ends of the machine. Many more sensors are often used to help in the balance calculations and confidence along the way by reviewing the projected vibration for each balance correction prior to making a shot.

The balance calculations are done using the same math approach as before, but expanded to accommodate the multiple plane data as shown below for a two plane correction with any number of vibration sensors:

The vibration vectors would also be arranged into a column matrix as before:

$$|V|_{run} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ \vdots \end{bmatrix} \text{ where } V_i \text{ is the vibration amplitude/phase at sensor } i \text{ for a given run}$$

The vibration change for all sensors and a single weight change is $\Delta V_{weight\ change} = |V|_k - |V|_j$ between two runs j and k. If two trial runs are used as required for a two plane balance, the change in vibration matrix will be as follows including two columns each corresponding to a shot:

$$|\Delta V| = \begin{bmatrix} \Delta V_{12} & \Delta V_{23} \end{bmatrix}$$

The vibration change matrix includes the vector change between the 1st and 2nd runs (column 1) and the 2nd and 3rd runs (column 2). Rows in this matrix correspond to individual sensors. If a two plane balance is done with 4 sensors, this matrix will be a 4x2 (4 rows x 2 columns).

The trial weight changes are documented for each balance run and arranged in a square matrix which will be a 2x2 for a two plane balance as follows:

$$|W| = \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix} \text{ where } W_{ij} = \text{weight change (vector difference) for run } i \text{ in plane } j$$

Once these matrices are assembled, the influence coefficients are calculated as ($|C|$ found by transposing the term on the left below). The C matrix includes columns that correspond to correction planes and rows that correspond to sensors.

$$|C|^T = |W|^{-1} \times |\Delta V|^T$$

This is a more general form of calculation for the influence coefficients that can be done for any combination of trial weight sets provided that the trial weight sets do not form a singular matrix. A singular matrix would be one that will not invert due to the determinant being zero, and will result when the two rows in the matrix are linear multiples of each other. For example, the following table shows examples of acceptable and unacceptable trial weight combinations for a two plane balance:

Plane 1	Plane 2	Comment
Trial Run 1 – 1 @ 0° Trial Run 2 – 1 @ 0°	Trial Run 1 – 1 @ 0° Trial Run 2 – 1 @ 0°	Not acceptable Same weights for both runs
Trial Run 1 – 1 @ 0° Trial Run 2 – 2 @ 0°	Trial Run 1 – 1 @ 0° Trial Run 2 – 2 @ 0°	Not acceptable Trial run 2 is a linear multiple of trial run 1
Trial Run 1 – 1 @ 0° Trial Run 2 – 1 @ 180°	Trial Run 1 – 1 @ 0° Trial Run 2 – 1 @ 180°	Not acceptable Trial run 2 is identical to trial run 1 except for phase change
Trial Run 1 – 1 @ 0° Trial Run 2 – 2 @ 180°	Trial Run 1 – 1 @ 0° Trial Run 2 – 2 @ 180°	Not acceptable Trial run 2 is a linear multiple of trial run 1 and same phase difference in both planes
Trial Run 1 – 1 @ 0° Trial Run 2 – 0 @ 0°	Trial Run 1 – 0 @ 0° Trial Run 2 – 1 @ 0°	Acceptable
Trial Run 1 – 1 @ 0° Trial Run 2 – 0 @ 0°	Trial Run 1 – 1 @ 0° Trial Run 2 – 1 @ 0°	Acceptable
Trial Run 1 – 1 @ 0° Trial Run 2 – 1 @ 0°	Trial Run 1 – 1 @ 0° Trial Run 2 – 1 @ 180°	Acceptable Identical to a static move and couple move

Once the $|C|$ and $|V|$ matrices are assembled, the final weights are calculated relative to any vibration reading set using the same approach as described for the single plane numerical method:

$$|M| = -[|C|^T |C|]^{-1} \times |C|^T |V_{ref}|$$

$|M|$ is the correction weight set (plane 1 and plane 2) that will produce the lowest combined vibration by minimizing the squares of each sensor value used in the assembly. These weights will be the weights to be ADDED relative to the weights that were installed when $|V_{ref}|$ was measured. At this point, the only difference between a “final” weight calculation and a “trim” run is whether the reference vibration used to calculate the correction weights is the actual original vibration (which will require removing trial weights added since the reference run) or the last measured vibration (which allows existing weights to stay in place and calculated weights are added to existing for a trim correction).

Errors in calculations and in response can always be minimized by using the lowest reference available. If a final weight reduces the vibration significantly but not adequately, then it would be preferred to leave the final weight in place and add a trim using the last run as the reference. If another weight is calculated relative to the original reference, the non-linearity resulting from the original higher vibration will continue to influence the calculations. If the response is characterized using the original vibration and the final weight and weights calculated from the final weight shot, a smaller trim weight will be determined and the non-linearity of the response will not influence the trim run response as much. My personal preference is to always use the last two runs to recalculate new influences and using the lowest reference available to increase the accuracy.

The vibration response can again be predicted using the reference vibration used in the calculation along with the influence coefficient matrix and the calculated correction weight column matrix.

$$|V_{projected}| = |V_{ref}| + |C||M|$$

Static and Couple Method

Adding trial weights to certain classes of machinery are usually done using combinations of static and couple (dynamic) corrections. This occurs for several reasons that are partly due to historical practice and partly due to ease of balance weight positioning. From a historical perspective, power generation equipment (turbines and generators) have fairly symmetric rotors and can often be reviewed using static and couple resolution of vibration vectors. When this can be done, the static and couple corrections can be calculated and applied using two separate single plane calculations. In addition, many power companies and equipment suppliers have large amounts of historical static/couple response data for broad families of turbines and generators.

This concept really falls apart for non-symmetric rotors and for flexible rotors that operate above one or more critical speeds since the modal response of the rotor will not consistently follow expected static and couple characteristics.

With current balance response calculation capabilities (balance programs) it makes sense to analyze unbalance in these types of rotors using two plane methods that allow simultaneous solving of both the static and couple response. For data input into a balancing program, the typical plane 1 and plane 2 correction planes would be used to input the static and couple corrections and/or trial weights. When this is done, the calculated weights would result in a static and couple shot opposed to separate weights in each correction plane.

To demonstrate the difference between using a two plane balance program using a static/couple weight addition and separate balance planes, the two trial runs below are identical and will produce the same calculation results:

Static/Couple Method:

Trial Run 1 – 100 gram @ 0° on drive end and 100 gram @ 0° on non-drive end (static move)

Trial Run 2 – 100 gram @ 0° on drive end and 100 gram @ 180° on non-drive end (couple)

Program input – Plane 1 = Static Plane 2 = Couple

Trial Run 1 - Plane 1 = 100 @ 0° Plane 2 = 0

Trial Run 2 - Plane 1 = 0 Plane 2 = 100 @ 0°

Calculated weights are added to the machine with Plane 1 added as static (same weight to both ends at same phase) and Plane 2 weight added as a couple (same weight to both ends with Non-drive end weight located 180° from the drive end couple weight).

Identical Weight Selection for 2 Plane Method:

Trial Run 1 – 100 gram @ 0° on drive end and 100 gram @ 0° on non-drive end (static move)

Trial Run 2 – 100 gram @ 0° on drive end and 100 gram @ 180° on non-drive end (couple)

Program input – Plane 1 = Drive End Plane 2 = Non-drive End

Trial Run 1 - Plane 1 = 100 @ 0° Plane 2 = 100 @ 0°

Trial Run 2 - Plane 1 = 100 @ 0° Plane 2 = 100 @ 180°

Calculated weights are added to the machine with Plane 1 added to the drive end correction plane and Plane 2 weight added to the non-drive end correction plane.

When input and used in the balance program in this way, either method will produce the same balance results.

Selecting Multiplane Trial Weights and Phase Angles

The same general rules will apply to selecting trial weights for multiplane problems, except that the trial weight amount is ½ the amount calculated based on the entire rotor weight since ½ is added to each balance plane.

Since the balance process is a bit more tricky with the multiplane approach, it is generally much more logical (and significantly easier to review on vector plots) to add trial weights to each correction plane one at a time. This allows easy calculation of the balance coefficients for each plane on all sensors directly using the response data. Yet in cases where the vibration is excessive, it may be wise to add a trial weight in one (or both) correction planes, and leave that weight in place if the vibration is lower followed by adding a single weight in one plane or the other to resolve the two plane response. If this is done, it will be very difficult to manually separate the response for hand calculation of the balance coefficients, yet the software by design will automatically separate them out into influence coefficients at each sensor for each plane.

The same methods described for single plane trial weights apply for two planes except that based on the total rotor weight the trial weight is divided by two so that only half of the rotor trial weight is located in a single plane. It is also common to use static/couple type shots as well depending on the initial reference vibration.

For high speed rotors that operate above a critical speed, the phase change during a coastdown can be used to help identify the most probable location of unbalance. In particular, as the rotor approaches the 1st critical speed, the phase lag between the heavy and high spots will increase from near zero to 90° at the critical speed then increase to near 180° well above. As the rotor approaches the 2nd critical speed (well above the 1st), the end of the rotor with the predominant unbalance will generally show a 180° phase shift back to near the low speed phase (well below the 1st critical) with a similar 90° shift as it approaches the 2nd critical. This type of response is shown in figures 14 & 15 which were recorded on a centrifugal compressor that had a damaged impeller on the discharge end. The compressor operates below the 2nd critical speed. The vibration was higher on the discharge end, but the presence of high unbalance on the discharge end was confirmed by the phase response during a coastdown showing the phase below both the 1st and 2nd critical to be at approximately the same phase near 90°. Inspection during overhaul showed severe balance piston seal rubs and a loose impeller on the last (discharge end) impeller.

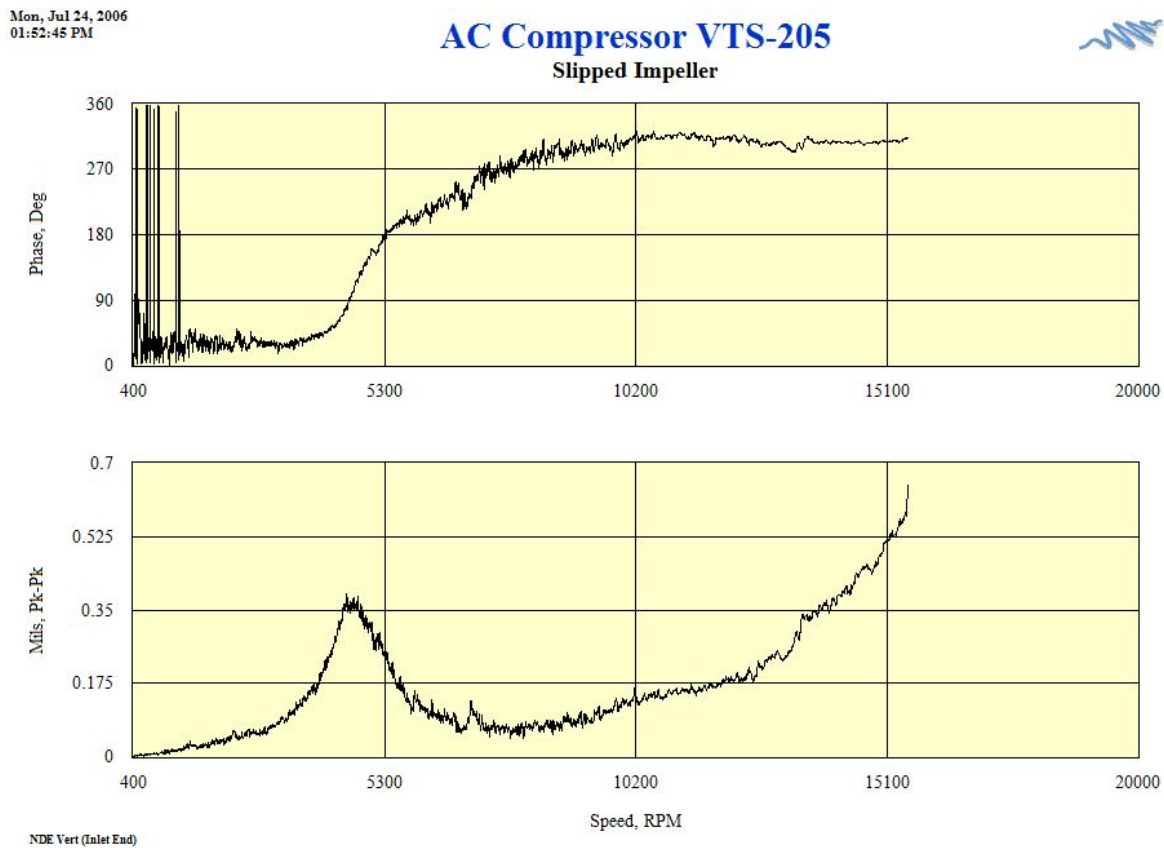


Figure 14 - Inlet End of Compressor (Side away from unbalance)

AC Compressor VTS-205 Slipped Impeller

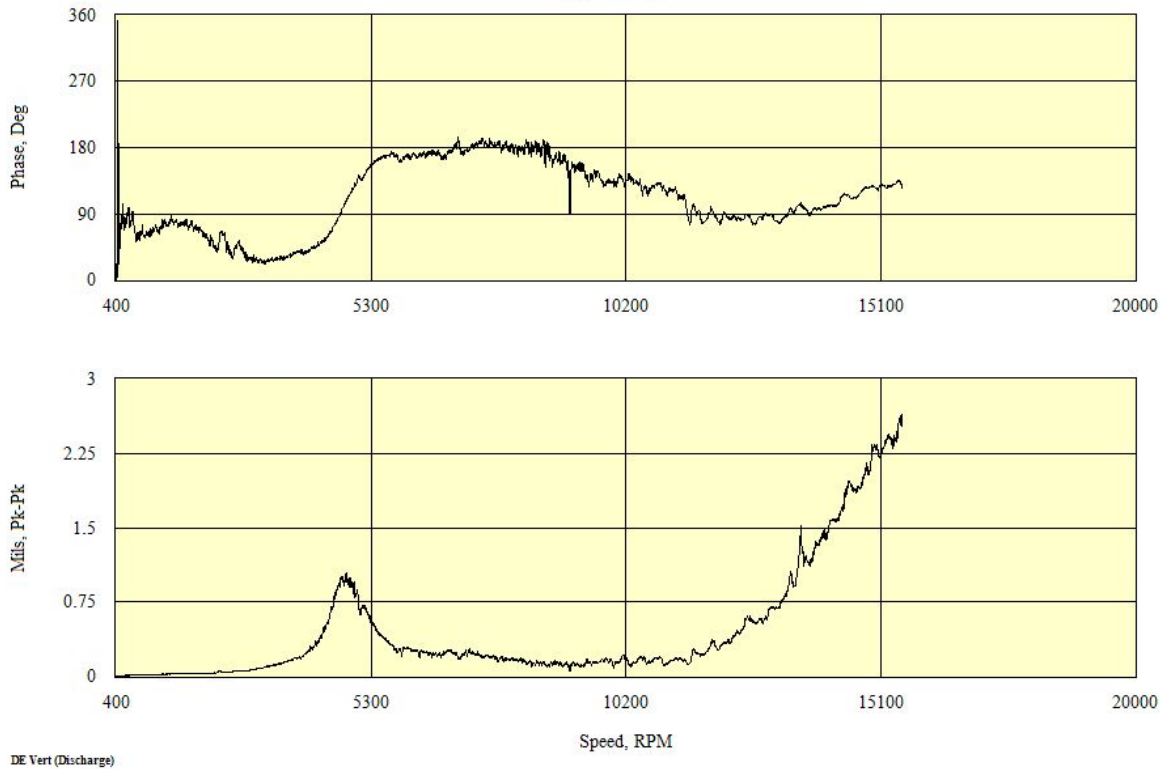


Figure 15 - Discharge End of Compressor (End with unbalance)

Summary

Methods have been described for completing field balancing using a combination of graphical and numerical methods. Application has been given for methods to eliminate the need for "taking a new O" which is a commonly taught process for dealing with balance non-linearity and data errors.

Theoretical description is also provided for calculating balance correction weights for single and multiple plane rotors. The method described is the least squares method which is used to minimize a number of sensors simultaneously whether using single or multiplane balancing.

Examples were presented to demonstrate the influence of non-linearity in the balance response as well as the influence of errors in weight placement. Methods were described to reduce the impact of data variations during a balance process. Emphasis on accuracy of weight addition angular position was made showing that a 5° angular position error is about equivalent to an 8% error in the amount of added weight.

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